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Longevity-linked life annuities: The Portuguese experience

Ana Catarina de Almeida Marques Amorim

Dissertation presented as partial requirement for obtaining the Master's degree in Statistics and Information Management, Specialization in Risk Analysis and Management

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by

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ABSTRACT

Over the last years, mortality rates have been declining, improving human population longevity. This exposes insurance companies and pension funds to longevity risk, which cannot be diversified since it affects all the population in the same direction. Furthermore, the longevity risk also has an impact on individuals that can outlive their assets in a context where the benefits provided by Social Security may not be enough.

As a result, several solutions have been proposed to cope with longevity risk, such as, the use of new approaches in terms of product design allowing either the creation of risk-sharing mechanisms between individuals and insurers or a complete transfer of the risk from insurers to individuals.

Considering that the longevity risk needs to be analyzed in accordance with the characteristics of a reference population, the recent improvements noted in the longevity of the Portuguese population, and, in order to take into account the policyholder perspective, this investigation is focused on the implementation, in terms of pricing, of two contract structures that are based on risk-sharing mechanisms, namely Longevity-index life annuities (LILA) and Longevity-contingent deferred life annuities (LDLA).

To perform this study, age-specific mortality rates were forecasted through the Poisson Lee-Carter model and financial returns were projected considering a Geometric Brownian motion and the 2-factors Vasicek model. Furthermore, considering that the benefits of a LILA can change overtime, it was also calculated an Utility-equivalent fixed life annuity through the use of a Constant Relative Risk Aversion model.

The results show that, when no limit is imposed on the risk transferred to the individuals, the price of a Longevity-index life annuity with profit share represents between 92.51% and 102.24% of the price of an immediate life annuity, which means that in a low-risk scenario, the losses caused by changes in longevity can be compensated by the investment returns present in the contract. Concerning the Longevity-contingent deferred life annuities, where the period of deferment changes with the dynamics of life expectancy, the results show that for the different ages, an additional period of deferment between 1 and 3 years would be needed to manage the longevity risk inherent to the contract.

Finally, it is concluded that the use of longevity-linked annuities provides a relevant solution to manage the insurers’ concerns with unexpected changes in life expectancy, while also gives the individuals a chance of acquiring an annuity product at a reduced price. However, the product design should be carefully studied by the insurers, considering the implementation of limits on the risk transferred to the policyholders in order to avoid issues in terms of demand.

KEYWORDS

Life Insurance; Longevity Risk; Annuity Market; Longevity-linked Annuities; Retirement

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LIST OF ABBREVIATIONS AND ACRONYMS

AIC	Akaike Information Criterion
ARIMA	Autoregressive Integrated Moving Average
CRRA	Constant Relative Risk Aversion
EA	Utility-equivalent level annuity income
EFLA	Utility-equivalent fixed life annuity
LDLA	Longevity-contingent deferred life annuity
LILA	Longevity-index life annuity
OLS	Ordinary least-squares
SVD	Singular Value Decomposition

1. INTRODUCTION

The mortality rates have been presenting decreasing trends over the last century with the improvements made in the living standards, medical progress, and population awareness concerning healthier lifestyles (Bravo, Corte Real & Silva, 2009).

A recent study (Foreman et al., 2018) developed a forecasting platform from which future health scenarios can be examined across countries and over time and concluded that, overall, health outcomes will show improvements until 2040. The same study refers to an improvement in the life expectancy for Portugal of 3.5 years by 2040, which is above what is forecasted for other countries, being highlighted that "among high-income countries, most saw forecasted increases of 1 to 3 years in life expectancy by 2040; an exception was Portugal, which had a projected gain of 3.5 years" (p. 2078).

The increasing life expectancy for the Portuguese population is also highlighted in the press release from the Portuguese Statistics Institute (INE - Instituto Nacional de Estatística, 2020) concerning the 2017-2019 Life Tables. In 2019, the life expectancy at birth for Portugal was estimated by the Portuguese Statistics Institute (INE, 2020) at 80.93 years, representing an increase of 1.99 years in the last decade, for the total population. Furthermore, the life expectancy at age 65 has also shown a significant improvement in the last 10 years with an increase of 1.22 years for men and 1.26 years for women.

Although positive, the foreseen increases in life expectancy come with consequences for both individuals and insurance companies. On one hand, the increases in life expectancy and the reduced Social Security benefits can expose individuals to the risk of outliving their assets (Olivieri & Pitacco, 2019). Indeed, for the Portuguese case, increasing life expectancy combined with changes in other demographic factors, such as the reduction of fertility rates, are forecasted to generate a declining working-age population and an increase in old-age dependency ratios, challenging the pension system's financial solvency (Bravo et al., 2012a,b, 2013, 2014; Moreira et al., 2019)

On the other hand, Olivieri and Pitacco (2019), also refer that the difficulties in predicting life expectancy can cause a reluctance in the insurance companies when providing life annuities due to the longevity risk.

Following Homa (2020), longevity risk can be divided into three components: i) the risk that someone will die earlier or later than expected (volatility risk); ii) the risk of the incorrect estimation of the current level of mortality for a given population (risk of mortality rate), and iii) the risk of the incorrect estimation of the future trend in mortality rate (risk of mortality rate trend).

The first two components are considered idiosyncratic (specific) risks meaning “that the people whose life expectancy exceeds that of the average can be balanced by those that do not live to the average” (Homa, 2020, p.26).

The last component of the longevity risk is the systematic risk that can be defined as “the risk that members of some reference population might live longer, on average, than anticipated” (Bravo, Corte Real & Silva, 2009, p.4).

The systematic longevity risk can have a crucial effect on life insurance portfolios since it impacts all individuals in the same direction, and therefore, cannot be diversified. Indeed, longevity risk was considered the main factor affecting life insurance portfolios in recent years (Bacinello, Millossovich & Chen, 2018).

Furthermore, within the Solvency II framework, the uncertainty associated with long-term guarantees, such as interest rate and mortality, can be costly considering that it leads to an increase of capital requirements (Bravo & El Mekkaoui, 2018).

As a result, it is essential to find solutions to cope with the systematic longevity risk. In this context, as stated in Olivieri and Pitacco (2019), several alternatives can be considered when managing systematic longevity risk, including (i) premium loadings; (ii) capital allocation; (iii) longevity-linked reinsurance, (iv) longevity-linked securities; (v) and through product design by creating mechanisms, such as longevity-linked annuities, allowing risk-sharing between insurers and individuals.

However, these solutions also carry some disadvantages as explained by Olivieri and Pitacco (2019) and by Bravo and El Mekkaoui (2018). In fact, an increase in the premium loadings or on the capital allocation can reduce the individuals’ or shareholders’ interest in annuity products. The reinsurance alternative tends to be expensive and there is a limited capacity in the market to cover exposures to longevity risk. The use of longevity-linked securities is still limited by an underdeveloped market, presenting several valuation issues. Finally, the use of longevity-linked annuities can present some issues for the individuals, since they can lead to a decrease in the benefits, causing a reduction in the demand.

Nevertheless, longevity-linked annuities have been appointed by several authors as an attractive solution for the issue presented with the possibility of creating a product that can bring a balance between the risk bared by the insurer and by the individual. From the insurer's perspective, with a reduction of the long-term responsibilities and thus a decrease of capital charges and, from the individual point of view, with a reduction of the annuity premiums' as compensation for the risk taken.

Furthermore, the use of annuities in the retirement phase is also proposed by OECD in a recent analysis of the Portuguese Pension system:

> "…Another important reform is to encourage at least partial annuitization in retirement. Portugal's annuity market is relatively underutilized, especially by members of voluntary personal pension funds. […] However, if the voluntary pension market gains prominence and average assets per individual grow, individuals could be encouraged to better utilize products such as annuities. This would allow them to smooth retirement income over their lifetimes." (OECD, 2019, p.130)

However, with the use of traditional solutions, the annuities market is currently unattractive, not only for insurers (as mentioned above) but also for individuals. In fact, despite the "strong theoretical and quantitative indications on the value of annuitization of wealth at the end of the lifecycle, the empirical evidence on the importance of and demand for private sector annuities in countries across the world remains very limited" (Holzmann, 2015, p.4).

The research performed by Holzmann (2015) explores the main reasons that explain the low levels of annuitization observed, namely: i) the existence of annuity alternatives in the presence of publicly provided retirement benefits; ii) bequest motives; iii) the incompleteness of the financial markets combined with the existence of several risks, such as the potential illiquidity in the annuity products; and finally iv) behavioral limitations including, the implicit complexity in annuity products combined with the existence of many options not easily compared and the individuals' loss aversion.

It is therefore important to find ways to improve the attractiveness of the annuities market in Portugal, both for providers and individuals, namely through the development of new products that can comprise, first, insurance-type products with risk-sharing mechanisms, where the abovementioned longevity-linked annuities are included; and second, non-insurance products where the risk is completely transferred to the individuals (Bravo, 2021). An overview of these two classes of products is given in the literature review.

At this stage, it is essential to highlight that the implementation of an insurance product must take into account the specificities of the market and, in the case of annuities, the mortality experience of the country. Thus, even with the existence of several studies performed that present an important basis for the application of new annuity products, it is important to perform an investigation considering the reality of the country, bearing in mind that "prescribing a generic model that will minimize the systematic risk and, in addition, fit all countries in all phases of development—and, hence, the losses for both the insurer and the insured—is impossible" (Alho, Bravo & Palmer, 2013, p.426).

In this thesis, two types of longevity-linked annuities were applied, in terms of pricing, to the Portuguese reality namely, Longevity-index life annuities (Denuit et al., 2011; Bravo & El Mekkaoui, 2018) and Longevity-contingent deferred life annuities (Denuit et al., 2015). For this purpose, age-specific mortality rates were forecasted through the Poisson Lee-Carter model and financial returns were projected considering a Geometric Brownian motion and the 2-factors Vasicek model. Furthermore, considering that in a LILA, the benefits can change overtime, for this product, it was also calculated an Utility-equivalent fixed life annuity using a Constant Relative Risk Aversion model.

As regards to the LILA, the main results show that when no limit is imposed on the risk transferred to the individuals, the price of an annuity with profit share represents between 92.51% and 102.24% of the price of an immediate life annuity, meaning that, in a low-risk scenario, for certain ages, the losses created by longevity changes are compensated by the financial returns included in the contract.

In terms of the utility to the policyholder, it was verified that for all of the low and medium risk-averse individuals', the EFLA values are always higher than the correspondent immediate fixed annuities. Still, for the high risk-averse individuals it was obtained the opposite result for most of the ages under analysis.

Concerning the LDLA, where the period of deferment varies following changes noted in life expectancy, considering a threshold defined at the contract inception, the results show that for the different ages and risk levels, the additional period of deferment, beyond the retirement age, needed to reach the life expectancy threshold defined ranges from 1 to 3 years.

In the end it was concluded that the use of longevity-linked annuities provides a relevant solution to manage the insurers' concerns with unexpected changes in life expectancy, while also gives the individuals a chance of acquiring an annuity product at a reduced price. Nevertheless, considering the risks taken by the individuals with a possible reduction of the benefits level or the extension of the deferment period, the implementation of caps should always be considered by the insurance companies to deal with eventual issues in terms of demand of this type of products.

The organization of the study is as follows: i) chapter 2 includes a literature review with an overview on the two types of products mentioned above (with and without risk-sharing mechanisms); ii) chapter 3 includes a description of the products selected for implementation under the Portuguese context, of the methodology adopted for the forecast of age-specific mortality rates and financial returns and of CRRA utility function considered; v) chapter 4 presents and discusses the results obtained; and finally, vi) chapter 5 presents the conclusions.

2. LITERATURE REVIEW

As abovementioned, one of the alternatives to face systematic longevity risk is the redefinition of the products available in the market through the creation of mechanisms that allow a risk transfer from the insurers to the individuals.

The risk transfer can be done considering two different approaches: i) first with a complete transfer of the risk to policyholders; ii) or, as an alternative, through a risk-sharing mechanism where only a part of the risk is borne by the individuals (Bravo, 2021).

2.1. PRODUCTS WITH A COMPLETE RISK TRANSFER TO INDIVIDUALS

The literature highlights several alternatives to fully transfer longevity risk from insurers to policyholders, namely through the use of pool arrangements where the risk is shared between individuals. As explained by Olivieri and Pitacco (2019), in this type of products:

> “..Liabilities must always be funded; this target is reached by letting the benefit amount decrease, if required by the available asset amount. Such arrangements rely on pooling arguments; they do not provide explicit longevity guarantees, and they are unable to absorb systematic losses caused by unanticipated mortality improvements.” (Olivieri & Pitacco, 2019, p.4).

Some examples of these products include: Group-Self Annuitization pools; Pool Annuity Funds and Mortality-linked funds; Annuity Overlay Funds; Tontines; and Tonuities.

Group-Self Annuitization pools are presented in Piggott, Valdez and Detzel (2005) as a product with no guarantees provided by the insurer and where specific longevity risk is shared within the pool and the systematic risk is borne by the individuals.

Donnelly, Guillén and Nielsen (2013) explore other options with a comparison between a Pooled Annuity Fund and a Mortality Linked Fund. In a Pooled Annuity Fund, individuals pool their mortality risk while maintaining investment freedom, which is one one the advantages of the product relative to Group-Self Annuitization. Mortality Linked Funds are similar to Pooled Annuity Funds but present a difference in what concerns the exposure to the mortality risk pooling. In a Mortality Linked Fund, “…there is no direct exposure to the pooling of mortality. Instead, the wealth of each fund member increases at a mortality-linked interest rate that is proportional to their force of mortality. The force of mortality is deterministic…" (Donnelly et al., 2013, p.68).

To increase the cost transparency in the annuity market, Donnelly, Guillén and Nielsen (2014), propose the use of an Annuity Overlay Fund that, as the products described above, has no guarantees and no protection against longevity risk. However, there is a crucial difference between an Annuity Overlay Fund and the products abovementioned: it is actuarially fair at any time. As a result, in this case, the participants can leave the fund at any time without the payment of a financial penalty. Furthermore, the product provides to each individual "a true individual investment freedom. They can decide at any time to change their investment strategy, again without paying any financial penalty" (Donnelly et al., 2014, p. 16).

Milevsky and Salisbury (2015) perform an analysis on Tontines with an overview of the historical context of the product and the study of its' utility in comparison with a conventional life annuity. A Tontine is also a type of fund where it is created a pool of annuitants that decide on a contribution scheme. Periodically, the value of the fund is distributed to surviving participants. As the number of participants decreases, there is an increase in the benefits distributed to the ones that are still alive.

The study of Tontines is also performed by Chen, Hieber and Klein (2019) that suggest a different approach with a combination between a Tontine and an annuity, creating a new product called Tonuity. In this case, for early ages of retirement, the product will have a behaviour equivalent to that of a Tontine. Later in life, with the increase of the participant's age, the payments scheme evolves towards a level deferred life annuity.

2.2. PRODUCTS INCORPORATING RISK-SHARING MECHANISMS

Annuity products are designed to protect against the longevity risk faced by individuals in the retirement phase of their lives. As explained by Bravo (2019), annuities can take several forms, being distinguished by the type of guarantees provided, frequency, duration and starting date of the payments.

Therefore, at this stage, it is essential to distinguish the longevity-linked annuities from the traditional annuity products usually provided by insurance companies.

Following Bravo and El Mekkaoui (2018), longevity-linked annuities present several characteristics that are also seen in a traditional annuity. However, this type of contract presents “a crucial difference: benefit payments are linked to a mortality index and, as such, leave annuitants’ incomes and consumption possibilities exposed to the uncertainty associated with the mortality risk of the group” (Bravo & El Mekkaoui, 2018, p. 213).

So, the longevity-linked annuities are insurance contracts where the benefits can be adjusted over time depending on the mortality experienced for a given portfolio, allowing risk-sharing between individuals and insurance companies.

The study of the longevity-linked annuities has been done for several authors in the last years with different approaches in what concerns the method used to adjust the benefits paid to policyholders over time.

Bravo, Corte Real and Silva (2009) used data for the Portuguese population from 1970 to 2004 to construct prospective life tables and developed a participating life annuity in which the risk is shared between the insurer and the individuals through the use of predefined thresholds in terms of the number of annuitants alive at a certain period time. With this method, when the number of people alive at the end of the period is higher than the defined threshold, the benefit will be proportionally reduced between survivors.

Also considering the use of a threshold, Denuit, Haberman and Renshaw (2015) proposed the application of longevity-contingent deferred life annuities. In this case, the payments are delayed for a period that can vary following a life expectancy threshold that is defined at the contract inception. In this case, the payments begin: (i) when the life expectancy is for the first time lower or equal to the threshold defined; or, (ii) at the end of the initial deferral period, at a smaller value.

Another approach considered in the literature is the use of a ratio between survival probabilities, as discussed in Denuit, Haberman and Renshaw (2011) that proposed the use of longevity-indexed life annuities, adjusting the benefits according to the ratio between the number of people who survived and the number of people that were expected to survive. Considering a similar approach, Bravo and El Mekkaoui (2018) studied the decomposition of a pure longevity-linked life annuity into a traditional fixed annuity and a basket of European-style longevity options and developed a study based on the French mortality. Both authors consider as an option the definition of a limit on the risk transferred to individuals, taking into account that, in some cases, those are not willing to take all the systematic longevity risk.

Alho, Bravo and Palmer (2013), suggested three models of variable annuities in public pension reform with an adjustment on the benefits performed every 5 years, following the updated projections for the cohort life expectancy, redistributing the account balance within the remaining survivors. The authors concluded that insurers would have lost money on fixed annuities whereas in variable annuities the losses could be minimized with the adjustment of the annuity benefits for higher ages.

Richter and Weber (2011) proposed a mortality-indexed annuity where the benefits are adjusted following a ratio between the initial reserves and the value of the reserves calculated at a certain future time for the individuals that are still alive, setting the benefits until the end of the contract, in case there are no further deviations on the estimated life expectancy.

Maurer et al. (2013) also considered a benefits adjustment process based on the reserves with the study of Variable Investment-Linked Deferred Annuities. This type of contract provides lifelong payments equal to the value of a prespecified number of units on a specific asset portfolio of mutual funds, where the annuitants can choose, with some limits, how the assets will be invested in terms of categories.

Finally, Olivieri and Pitacco (2019) gathered the different alternatives proposed by the literature and presented a general framework with the analysis of several benefit structures, including: i) linking benefits to the survival rate, with target probability set at the beginning of the contract; ii) linking benefits to the survival rate, with target probability that can be subject to update after the contract issuance; iii) linking benefits to the actuarial value of the annuity, with target life table set at the beginning of the contract, to be compared to the latest life table; iv) and linking benefits to the actuarial value of the annuity, with target life table set after the contract issuance, to be compared to the latest life table. In the end, Olivier and Pitacco (2019) concluded that the more reasonable alternative for both insurers and individuals would be the link between the benefits and a survival rate with a threshold defined at the beginning of the contract.

For public annuities, in recent decades, most high-income countries have responded to continuous life expectancy increases, below replacement-level fertility, an upward trend in old-age dependency ratios, low productivity gains and economic growth, a rapidly shifting labour market and declining financial market returns with systemic (e.g., the switch towards a Non-Financial Defined Contribution (NDC) scheme in Sweden, Italy, Poland, Latvia and Norway; pension financialisation, i.e., the expansion of private complementary occupational and personal pre-funded defined-contribution (DC) pensions) and/or gradual parametric public pension reforms (e.g., updates in the early and normal retirement ages, modifications in the defined benefit (DB) pension formula) as part of their efforts to reduce or eliminate short-term and long-term imbalances between revenues and expenditures, alleviating the pressure on public finances, together with efforts to preserve minimum pension adequacy.

A common denominator in most pension reforms adopted in developed countries has been to automatically link pension benefits to life expectancy developments observed at retirement ages. The link has been established and reinforced in multiples ways (Ayuso et al., 2021a,b; Bravo & Herce, 2020; Bravo & Ayuso, 2020, 2021; Bravo et al., 2021): (i) by indexing normal and early retirement ages to life expectancy (e.g., Denmark, The Netherlands, Portugal, UK, Slovakia , Italy, Finland); (ii) by linking initial pension benefits to sustainability (also called life expectancy) factors (e.g., Finland, Portugal, Spain), to old-age dependency ratios (e.g., Germany, Japan) or to life annuity coefficients (e.g., Sweden, Italy, Poland); (iii) by indexing the eligibility requirements to the contribution length (e.g., France, Italy); (iv) by conditioning the annual pension indexation (e.g., The Netherlands, Luxembourg); (v) by conditioning the pension penalties (bonuses) for early (late) retirement to the contribution length (e.g., Portugal); (vi) by introducing longevity-linked risk-sharing life annuities in public and private pension schemes (e.g., The Netherlands, USA) ; (vii) by phasing in national Financial Defined Contribution (FDC) plans (e.g., Chile); (viii) by determining the accumulation of pension entitlements (e.g., The Netherlands).

Automatic pension stabilizers modify the redistribution of costs and benefits within and across generations (e.g., between current and future pensioners, between the lifetime rich and the lifetime poor) and alter the way risk is shared within and across generations, for instance, by changing the way individual and aggregate longevity risk are pooled among plan participants, by transferring interest rate and investment risks to retirees (Barr & Diamond, 2009). They also change the nature of the pension promise that is being offered to younger workers and represent a paradigm shift in the responsibility for old-age income, in what some authors name the (risk) privatization and marketization (commodification) of pension policy.

3. MATERIALS AND METHODS

To consider the point of view of the insurers and individuals, it was decided to advance with the analysis of contracts that are based on risk-sharing mechanisms. Therefore, two types of contract structures/products were selected to test their impact on the Portuguese context in terms of pricing. The products selected include: (i) Longevity-index life annuities (Denuit et al., 2011; Bravo & El Mekkaoui, 2018); and (ii) Longevity-contingent deferred life annuities (Denuit et al., 2015).

Furthermore, this selection allows the study of two different approaches considered in the literature to manage the systematic longevity risk, namely the use of a survival ratio and the use a deferment period.

To proceed with the implementation of these solutions it was defined a methodology that is based on three steps: (i) forecasting of age-specific mortality rates; (ii) the projection of financial returns; (iii) and finally, considering that under a Longevity-index life annuity benefits can be adjusted during the contract, the calculation of an utility-equivalent fixed life annuity (EFLA) for this type of product.

3.1. THE CONTRACT STRUCTURES

3.1.1. Longevity-index life annuities

As explained above, the first model selected is a Longevity-index life annuity, as presented by Denuit et al. (2011) and Bravo and El Mekkaoui (2018). In this type of contract, each individual will receive an annual payment if he or she is still alive at a certain age. However, its' amount depends both on a longevity index and on an interest rate adjustment factor.

Here it is assumed that, an individual age x_0 buys an annuity contract at year t_0 with an amount of one monetary unit scaled by a longevity index. Let ${}_{k}p_{xo}^{[\mathcal{F}_0]}(t_0)$, k = 0,… ,$\omega - x_0$ be the forecasted probability of an individual, of a certain reference population, aged x_0, in year t_0, being alive at age $x_0 + k$ and ω the highest attainable age.

One year later, the actual number of people that survived from the reference population, ${}_{k}p_{xo}^{[\mathcal{F}_k]}$, becomes available. As a result, the longevity index is calculated by comparing the forecasted number of survivors with that observed at the end of the period:

$$I_{to+k} = \frac{{}_{k}p_{xo}^{[\mathcal{F}_0]}(t_0)}{{}_{k}p_{xo}^{[\mathcal{F}_k]}(t_0)} = \prod_{j=0}^{k-1} \frac{{}_{k}p_{xo}^{[\mathcal{F}_0]}(t_0+j)}{{}_{k}p_{xo}^{[\mathcal{F}_k]}(t_0+j)}. \tag{1}$$

Concerning the interest rate adjustment factor, let R_t be the observed net investment return in year t and i be the guaranteed interest rate. The k-year interest rate adjustment index for, k = 1,…,$\omega - x_0$ follows the formula below:

$$R_{to+k} = \prod_{t=1}^{k} \frac{(1+R_t)}{(1+i)}. \tag{2}$$

Based on the longevity index and interest rate adjustment factors defined above, assuming that each annuitant has the same level of contributions and receives an equal benefit, the annual benefit (b_t) will be adjusted as follows:

$$b_{t0+k} = b_{t0} \times I_{to+k} \times R_{to+k}. \tag{3}$$

Therefore, under the presented mechanism, the premium of a Longevity-index life annuity is defined as:

$$a_{x0}^{LILA}(t_0 | \, I_{to+k}, R_{to+k}) = \sum_{k=1}^{\omega - x_0} Z\,(0,k) .\, {}_{k}p_{xo}^{[\mathcal{F}_k]}(t_0) . b_{t0} . I_{to+k} . R_{to+k}, \tag{4}$$

where $Z\,(0,k)$ is the discount factor (the price of a zero-coupon bond with face value one monetary unit) starting at time 0 and maturing at time k using a given discount spot market interest rate.

3.1.1.1. Capped longevity-index life annuity

Considering that individuals present different levels of risk aversion, it can happen that an annuitant is not willing to support all longevity risk. In fact, "…if the annuitants absorb all of the systematic risk, annuity payments may become arbitrarily low in old ages in the case of adverse experience. This situation appears to be highly undesirable given that longevity insurance is the main purpose of annuities…" (Denuit et al., 2011, p. 103).

Thus, Denuit et al. (2011) and Bravo and El Mekkaoui (2018) also consider the use of a capped longevity-index life annuity where a part of the longevity risk is borne by the insurer.

In this case, the longevity index is capped with the definition of lower and upper limits that can either be constant or change during the contract, considering that

$$0 < I_{t0+k}^{min} < 1 < I_{t0+k}^{max}, \tag{5}$$

where, I_{t0+k}^{min} is the lower limit in terms of guarantees and I_{t0+k}^{max} is the upper limit in terms of the annuity benefits.

Therefore, when calculating the premium (best estimate of the fair value) of a capped longevity-index life annuity, the longevity index considered before should be replaced by:

$$I_{t0+k}\left(I_{t0+k}^{min}, I_{t0+k}^{max}\right) = \max\ \{\min(I_{t0+k}, I_{t0+k}^{max});\ I_{t0+k}^{min}). \tag{6}$$

3.1.2. Longevity-contingent deferred life annuities

Concerning the Longevity-contingent deferred life annuities presented in Denuit et al. (2015), it is considered an individual of age x_0 that buys an annuity contract at year t_0. However, unlike the previous model, this contract includes a deferment period, d. Therefore, the annuitant will only receive an annual payment in case he/she is alive at age $x_0 + d$.

Another important feature is that the deferment period is not fixed and changes following longevity dynamics. Here, the payments are delayed if at age $x_0 + d$ the life expectancy calculated at calendar year $t_0 + d$ exceeds a contractual value, e^* , specified at contract inception.

As a result, following the framework developed by Denuit et al. (2015) when the first period of deferment ends, there are three options:

- The life expectancy observed, e_{x0+d} , is smaller than the threshold defined and the payments can start, formally:

$$e_{x0+d}(t_0 + d) \leq e^* \tag{7}$$

- The life expectancy observed exceeds the threshold: $e_{x0+d}(t_0 + d) > e^*$. Thus, the payments are delayed for an additional number of years, Δ, starting at age $x_0 + d + \Delta$, where Δ should satisfy the following condition:

$$\Delta = \inf\left\{k \in \{1, 2, 3, \dots\} | e_{x0+d}(t_0 + d) \leq e^*\right\} \tag{8}$$

- The life expectancy observed exceeds the threshold ($e_{x0+d}(t_0 + d) > e^*$), but the annuitant is not willing to wait and decides to start the payments. Here the benefit payments will start at $x_0 + d$, however, at a reduced level in accordance with the factor below:

$$\frac{e^*}{e_{x0+d}(t_0 + d)} \tag{9}$$

In line with what was stated in the previous chapter for the Longevity-index life annuities, the annuitant may not be capable of supporting all of the longevity risk, so one of the alternatives would be to limit the risk transferred by defining a maximum value for the additional delay, Δ^{max} (Denuit et al., 2015).

3.2. THE MORTALITY PROJECTION MODEL

To generate forecasts for mortality rates, stochastic mortality models must be used. The standard approach to age-specific mortality rate forecasting is to pursue a “winner-take-all” perspective by which, for each population, a single believed to be «best» or «true» model is selected from a set of candidate approaches using some method or criteria, often neglecting model uncertainty (conceptual uncertainty) for statistical inference purposes. The actuarial and demographic literature proposes a vast number of single and multi-population discrete-time and continuous-time stochastic mortality models to forecast age-specific mortality rates (Bravo & Nunes, 2021).

Cairns, Blake and Dowd (2008) reviewed the different approaches considered in the literature regarding mortality modelling, explaining that for both discrete and continuous-time models, the majority of the researches have been focused on short-rate models but also refer the use of other approaches such as: i) P-splines and discrete-time market models, in what regards to discrete-time models; ii) and methods similar to forward-rate models and market models in interest-rate modelling, in what concerns the continuous-time models.

Furthermore, in their research Cairns et al. (2008), explain that the publication of the mortality data organized on an annual basis and by age can encourage to the adoption of this type of approach, highlighting five discrete-time models:

- The Lee and Carter (1992) model, which is one of the most popular approaches when modelling mortality and has been studied and adapted by several authors such as Brouhns, Denuit and Vermunt (2002);
- Multifactor age-period models, as proposed by Renshaw and Haberman (2003) and by Cairns, Blake and Dowd (2006);
- The Renshaw-Haberman cohort model (Renshaw & Haberman, 2006);
- The Cairns-Blake-Dowd model with a cohort effect, which is a generalization of the Cairns-Blake-Dowd two-factor model in Cairns, Blake and Dowd (2006);
- P-splines as discussed for example in Currie, Durban and Eilers (2004).

Considering that the Lee-Carter model has been commonly applied by different authors, actuarial groups and government statistical agencies (Alho et al., 2013), for the completion of this research we decided to adopt this model. Thus, the projection of mortality was performed through the Lee-Carter model, considering the Poisson modelling approach as proposed by Brouhns et al. (2002), that was developed in other to overcome the issues with the homoskedastic assumption for the error term in the original model, as explained further.

After the initial projection of mortality, the prospective life tables were completed through the use of the method developed by Denuit and Goderniaux (2005).

The data used in the estimation was obtained from the Human Mortality Database[1] and includes deaths and exposures-to-risk for Portugal from 1940 to 2018.

3.2.1. Poisson Lee-Carter Model

Lee and Carter (1992) developed a model for the projection of mortality through the use of a relationship between a log-bilinear form for the mortality force, $\mu_x(t)$, and three parameters, that is defined as follows:

$$\ln \widehat{\mu_x(t)} = \alpha_x + \beta_x \kappa_t + \varepsilon_x(t), \tag{10}$$

where,

α_x represents the average of $\ln \widehat{\mu_x(t)}$ overtime, so the general shape of mortality,

β_x represents the age-specific patterns of mortality change, indicating the sensitiveness of $\ln \widehat{\mu_x(t)}$ to changes in the time trend,

κ_t represents the time trend,

$\varepsilon_x(t)$ is the error term and represents the age-specific influence in mortality not captured by the model.

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[1] https://www.mortality.org/

Furthermore, the model parameters are subject to the following identification constraints:

$$\sum_t \kappa_t = 0 \ \ and \ \sum_x \beta_x = 1. \tag{11}$$

In the original framework of this model, the estimation of the parameters is done through an OLS (ordinary least-squares) approach using a singular value decomposition (SVD). However, the use of an OLS estimation requires a homoskedastic assumption for the error term.

Following Brouhns et al. (2002), this is an unrealistic assumption considering that “the logarithm of the observed force of mortality is much more variable at older ages than at younger ages because of the much smaller absolute number of deaths at older ages” (p. 378).

To avoid the issues with the OLS assumption in the original model, Brouhns et al. (2002) proposed a new framework based on a Poisson modelling that is defined as follows:

$$D_{xt} \sim Poisson\left(E_{xt}\mu_x(t)\right) \ \ with \quad \mu_x(t) = exp(\alpha_x + \beta_x\kappa_t), \tag{12}$$

where, D_{xt} represents the number of deaths recorded at age x during year t, from an exposure-to-risk E_{xt}.

In this approach, the parameters are still subject to the constraints referred above (11) and the estimation of the parameters is made by maximizing the log-likelihood given by:

$$L(\alpha, \beta, \kappa) = \sum_{x,t}\{D_{xt}(\alpha_x + \beta_x\kappa_t) - E_{xt}exp(\alpha_x + \beta_x\kappa_t)\} + constant. \tag{13}$$

Furthermore, it is considered the use of an iterative method for estimating log-linear models with bilinear terms (Goodman, 1979 as cited by Brouhns et al.,2002).

The estimation of κ_t is done through the methodology proposed Lee and Carter (1992) that starts with the identification of the appropriate ARIMA (autoregressive integrated moving average) to model the time series.

The selection of the ARIMA model, follows a Box-Jenkins method that is based on three steps: i) the identification that determines the order of the model (p, d and q) and is done through the construction of plots of data and of autocorrelation and partial autocorrelation functions; ii) the estimation of the model parameters through the use of maximum likelihood methods; iii) the diagnosis that is performed to ensure the adequacy of the model estimated.

3.2.2. Completion of the life tables

Due to the lack of data for older ages caused by small exposures, the Lee-Carter model can lead to inaccurate results when capturing the behavior of the force of mortality.

To overcome this issue, Denuit and Goderniaux (2005) developed a method that considers the use of a log-quadratic form able to capture the slower increases of mortality at older ages. The method proposed is as follows:

$$\ln \widehat{q_x} = a_t + b_t x + c_t x^2 + \varepsilon_t , \qquad \varepsilon_t \sim N(0, \sigma^2) \tag{14}$$

with the constraints,

$$q_{x_{max}} = 1 \; and \; q'_{x_{max}} = 0 \tag{15}$$

where q_x is the probability that an x-aged individual dies before reaching age $x + 1$.

3.2.3. Estimation of the market price for the longevity risk: Wang transform

To implement the contract structures under analysis, it was necessary to estimate the survival probabilities with the addition of price for the longevity risk. Following the approach suggested by Wang (2000), this estimation can be obtained through the application of a distortion operator:

$$\mathrm{g}_\lambda(u) = \Phi[\Phi^{-1}(u) + \lambda], \tag{16}$$

where,

$\Phi(u)$ is the cumulative standard normal distribution,

u is a probability, in this case, the best estimate of the survival probability obtained from the Poisson Lee-Carter Model,

λ is a parameter that defines the market price of risk.

3.3. THE PROJECTION OF FINANCIAL RETURNS

For the projection of the financial returns, it was assumed that the insurance company owns a portfolio composed of two instruments, stocks and bonds, with an allocation of 30% and 70% respectively to ensure that the portfolio can cover the responsibilities towards the policyholders (Bravo & El Mekkaoui, 2018).

With this assumption, the financial returns were estimated considering that stock prices follow a Geometric Brownian Motion defined as:

$$dS = \mu S dt + \sigma S dz, \tag{17}$$

where,

S is the market price of the underlying asset,

μ is a drift/trend of the expected return of the asset,

σ is the volatility, defined as the standard deviation on the asset return,

dt is the time increment,

dz is a Wiener process with expected return equal to zero and variance equal to dt, where $dz = \varepsilon\sqrt{dt}$, $\varepsilon \sim N(0,1)$.

Regarding the bond prices, to consider the possibility of different shifts in the yield curve, it was used a two-factor Vasicek (1977) model, where the spot rate $r(t)$ follows the sum of two Ornstein-Uhlenbeck processes as presented below:

$$
\begin{gathered}
r_t = x_t + y_t \\
dx_t = \beta_x(\mu_x - x_t)dt + \sigma_x dz_1 \\
dy_t = \beta_y\left(\mu_y - y_t\right)dt + \sigma_y dz_2
\end{gathered}
\tag{18}
$$

where,

$\beta(\mu - x)$ is the instantaneous drift of x, with $\beta > 0$,

μ is the long-term mean,

σ^2 is the constant instantaneous variance,

(dz_1, dz_2) is a two-dimension Brownian Motion with an instantaneous correlation of ρ.

To produce financial returns estimates, we performed a simulation of 10,000 trajectories with the data on: i) historical data on S&P prices from January 2000 to December 2020[2]; ii) monthly bond yields for the US 3-months and 10-year maturities[3].

The estimated parameters for the Geometric Brownian Motion and 2-factor Vasicek model are presented in Table 1 and Table 2.

μ	σ
0.0647	0.1993

Table 1 – Geometric Brownian Motion parameters

	μ	β	σ	ρ
Short Rate x	0.0155	0.3068	0.0003	0.7631
Long Rate y	0.0330	0.6658	0.0006	

Table 2 – 2-factor Vasicek model parameters

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[2] Source: Yahoo Finance (https://finance.yahoo.com), accessed on 28/03/2021

[3] Source: Yahoo Finance (https://finance.yahoo.com), accessed on 28/03/2021

3.4. THE UTILITY-EQUIVALENT FIXED LIFE ANNUITY

As explained before, the use of longevity-index life annuities assumes an adjustment on the benefits paid to policyholders that can have an impact on the demand of the product. Therefore, it is important to analyze this product from the annuitant perspective, which can be done through the use of an utility-equivalent.

To obtain the referred utility equivalent, it was considered a standard time addictive constant relative risk aversion (CRRA) utility function that takes into account the level of risk aversion and also the time preference of the individual. As explained in Bravo (2021), the CCRA utility function defined over consumption is as expressed as follows:

$$V_t = max_{C_t,C_{t+1},\dots,C_{\omega-x}} E_t \left[\sum_{k=t}^{\omega-x} \beta^k \left\{ {}_t p_x \frac{C_t^{1-\gamma}}{1-\gamma} \right\} \right] \quad ,\beta > 0, \gamma \neq 1, \gamma > 0, \tag{19}$$

where,

V_t is the expected lifetime utility,

γ is the coefficient of relative risk aversion,

β is the subjective discount factor.

To implement this function, it was assumed that the consumption level is equal to the benefit obtained through the longevity-index life annuity.

Then the CCRA utility function defined over consumption that was transformed into a utility-equivalent level annuity income, EA, (Mitchell et al., 1999, as cited by Bravo, 2021):

$$EA_t = \left[\frac{V_t(1-\gamma)}{\sum_{t=1}^{\omega-x} \beta^t \, {}_t p_x} \right]^{\frac{1}{1-\gamma}}, \beta > 0, \gamma \neq 1, \gamma > 0 \tag{20}$$

To end the analysis, through the use of the EA_t it was calculated the utility-equivalent fixed life annuity (EFLA) that would deliver the same utility as the longevity-index life annuities under study.

4. RESULTS AND DISCUSSION

Following the methodology described, this section includes the presentation and discussion of the results obtained in the projection of age-specific mortality rates and annuity premiums for the contract structures selected, as well as, the analysis of the utility-equivalent annuity for the longevity-indexed life annuities.

The calculations were performed using the R software (version 3.6.1).

4.1. RESULTS ON THE MORTALITY PROJECTION

As referred above, the data used in the mortality rates estimation was obtained from the Human Mortality Database[4] and includes deaths and exposures-to-risk for Portugal from 1940 to 2018.

The analysis performed was focused on the Portuguese mortality data for the ages between 50 and 95. The Figure 1 presents the evolution of the observed log mortality rates over the years with a decreasing trend that is visible for the different ages under the study.

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[4] https://www.mortality.org/

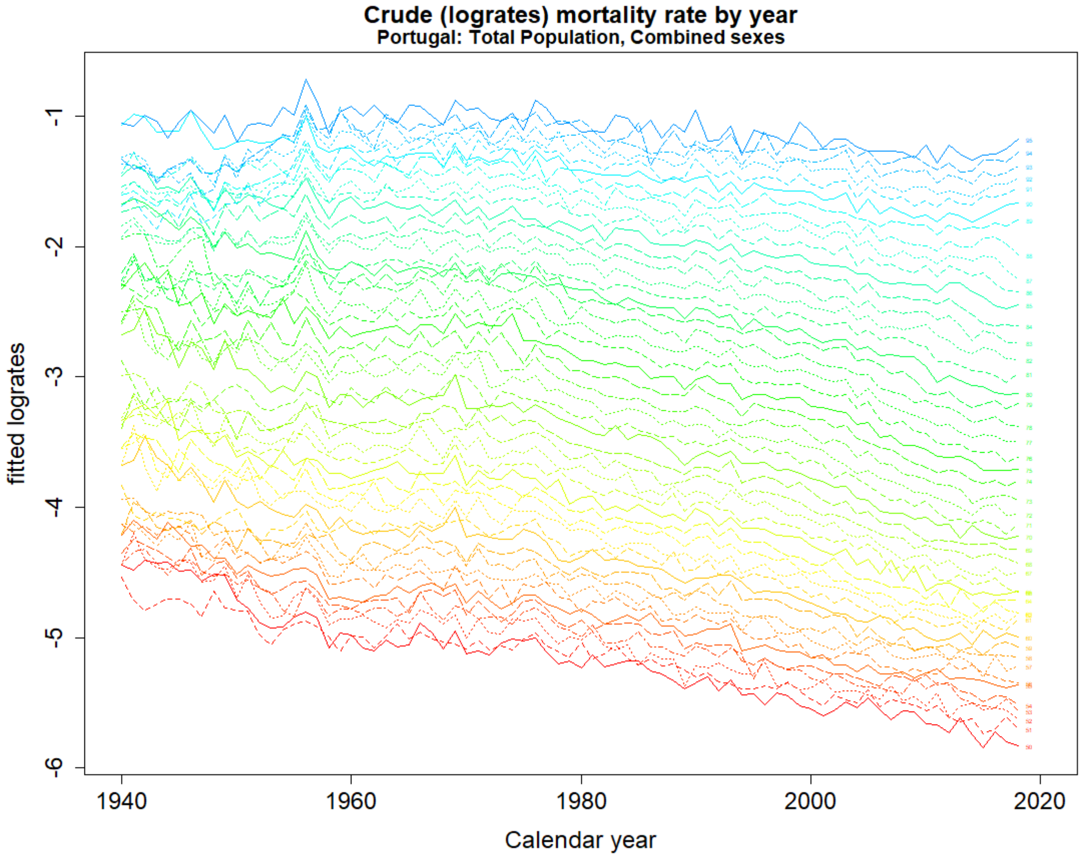

Figure 1 – Portuguese mortality rates by year: 1940 to 2018.

Source: Author's elaboration.

Moving to the mortality projection, Figure 2 presents the estimates obtained for the Poisson Lee-Carter parameters, with the general shape of mortality (α_x) growing overtime for the different ages and a decreasing sensitiveness of the mortality patterns to the time trend.

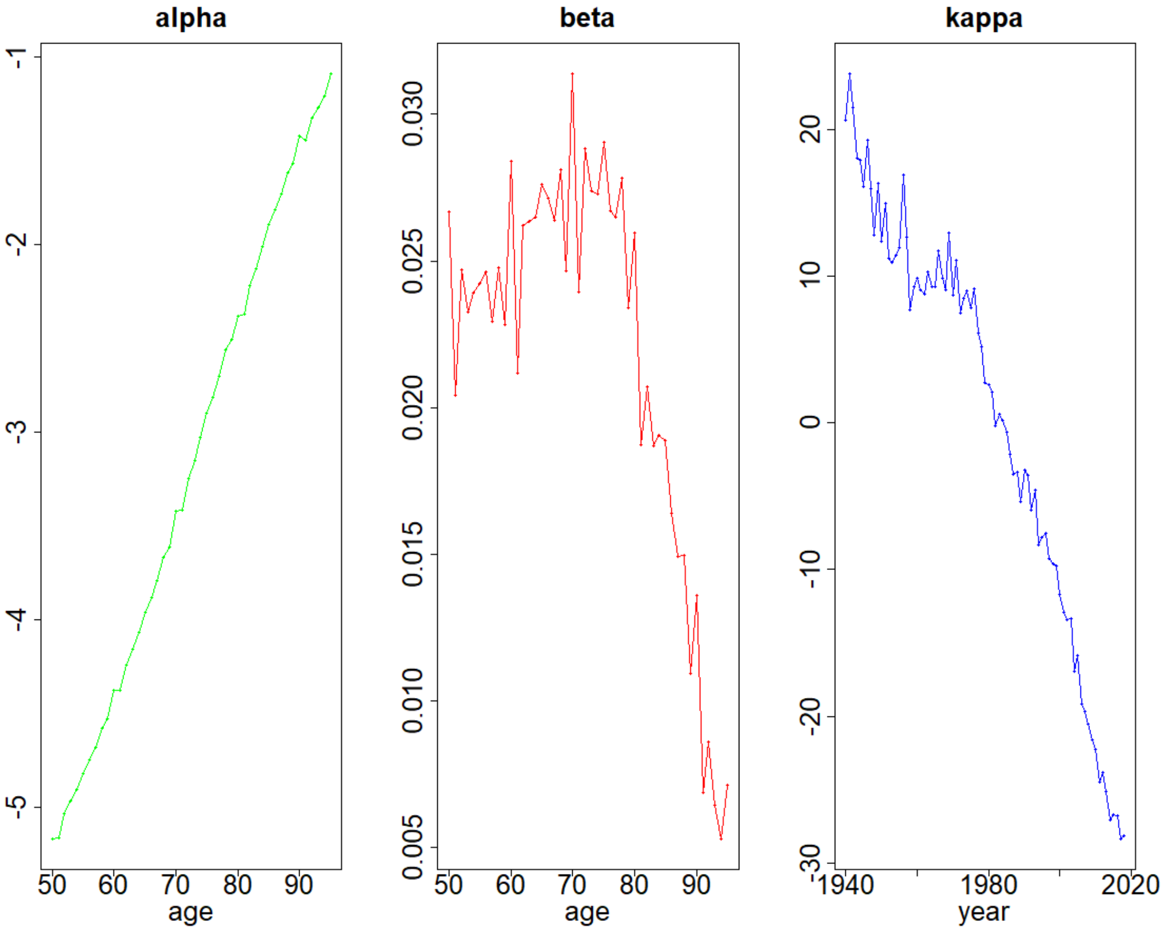

Figure 2 – Poisson Lee-Carter estimated parameters.

Source: Author's elaboration.

To complete the Poisson Lee-Carter projections, the time trend was estimated considering an ARIMA (0,1,5) with a drift, that is defined as follows:

$$\kappa_t = \mu + \varepsilon_t + \beta\varepsilon_{t-1} + \beta\varepsilon_{t-2} + \beta\varepsilon_{t-3} + \beta\varepsilon_{t-4} + \beta\varepsilon_{t-5} + \kappa_{t-1} \qquad (21)$$

$$\kappa_t = -0.6519 + \varepsilon_t - 0.5836\varepsilon_{t-1} - 0.0083\varepsilon_{t-1} + 0.1286\varepsilon_{t-1} - 0.1927\varepsilon_{t-4} + 0.3407\varepsilon_{t-5} + \kappa_{t-1}$$

$$\varepsilon_t \sim N(0, \sigma^2)$$

To select the appropriate ARIMA model, different alternatives were compared using the results obtained for the correspondent AIC (Akaike Information Criterion). Overall, the models with a drift presented a lower AIC in comparison with the ones without a drift.

Table 3 presents a sample of the ARIMA models compared in the process and the correspondent AIC. In the end, the ARIMA model selected was the one with the lower AIC.

Model	**AIC**
ARIMA (0,1,0) with drift	336.852
ARIMA (0,1,1) with drift	317.578
ARIMA (0,1,2) with drift	318.864
ARIMA (0,1,3) with drift	320.177
ARIMA (0,1,4) with drift	321.888
ARIMA (0,1,5) with drift	**316.078**

Table 3 – Sample of the ARIMA models tested

Source: Author's elaboration.

To ensure its' adequacy, the ARIMA model selected was tested to verify the normality of the residuals, through the performance of the Shapiro-Wilk and Ljung-Box tests and considering a significance level of 0.05. Both tests presented a p-value higher than 0.05.

Finally, as referred to in 3.2.2, the mortality projections were concluded with the application of the Denuit & Goderniaux (2005) method considering 130 years as the highest attainable age.

4.1.1. Results on the simulated mortality – application of the Wang Transform

As referred in 3.2.3, to incorporate the market price of longevity risk in the survival probabilities, it was applied of a distortion operator as defined in Wang (2000). The calculations performed considered values of λ between 0.1 and 0.3 (Lin & Cox, 2005 as cited by Bravo & El Mekkaoui, 2018). Figure 3 presents the results obtained for the year 2019, in terms of the log mortality rates by age.

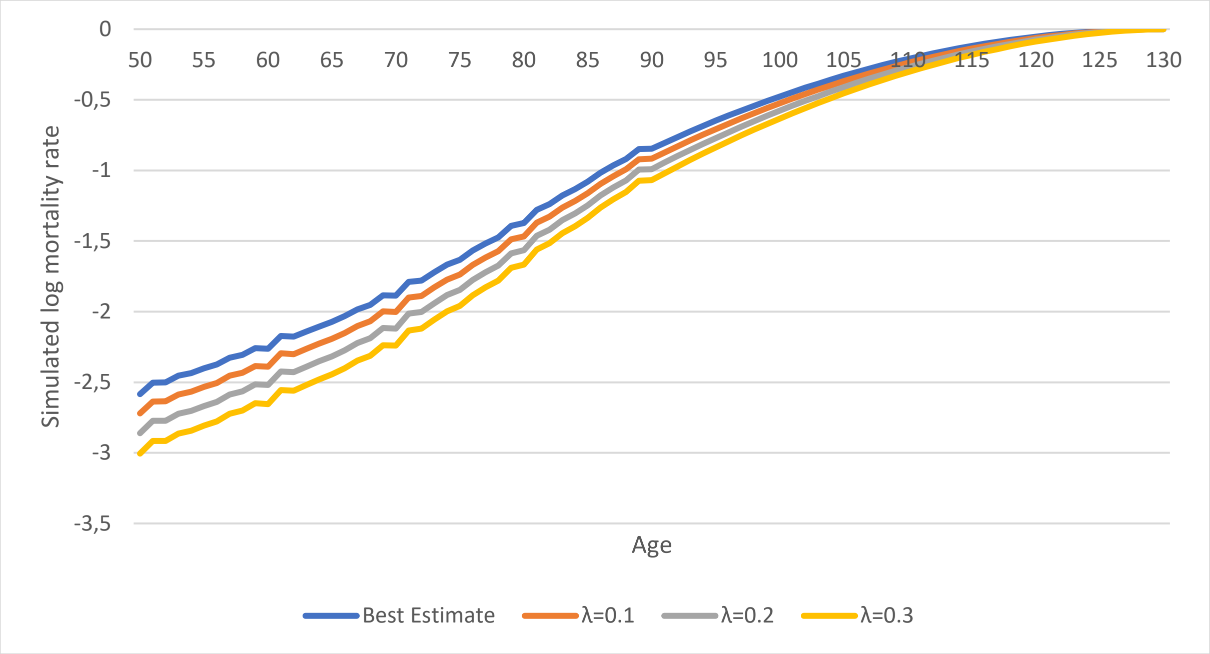

Figure 3 – Simulated Log mortality rates by age for the year of 2019

Source: Author's elaboration.

As expected, the changes in the risk parameter λ result in decreases in the mortality rates estimated for the different ages, presenting a higher impact for the younger ages.

To avoid the risk of underestimating longevity, all the calculations in this investigation were performed considering a cohort approach as a way of capturing changes in the life expectancy over time for the different ages.

4.2. ANNUITY PRICING RESULTS

4.2.1. Longevity-index life annuities

This section presents the annuity premiums obtained regarding the Longevity-index life annuities for ages between 50 and 75 years, including the impacts of changes in the risk parameter λ, which reflects the market price of longevity risk. Considering the current market conditions and the low levels of guarantees provided by insurers, all the calculations were performed considering an interest rate of 0%.

The LILA premiums are presented in two different approaches: i) the first approach without a limitation of the risk transferred to the annuitant (Table 4); and, ii) a second approach with a limit (cap) on the risk bared by the annuitant (Table 5). Regarding the second approach, two analyses were performed, assuming that the longevity-index, I_{to+k}, was limited either to 90% or to 80%, meaning that the benefits paid to policyholders could only be reduced by 10% or 20%, respectively.

The first approach (Table 4) is presented considering an alternative where there the financial results are not shared with the policyholder and another alternative where they are considered in the price to allow an analysis of the impact of this feature.

For comparison matters, it is also present the price of a regular immediate fixed annuity.

As expected, the LILA premiums show a decreasing trend as age grows, reflecting the smaller life expectancy for older individuals. Furthermore, as λ increases, the prices of the annuities decrease, reflecting the higher levels of risk transferred to the policyholder.

Concerning the results in Table 4, it can be noted that for the Longevity-Indexed Life Annuities without a profit share mechanism, the annuity prices represent between 87.33% and 97.23% of the price of an immediate fixed annuity. Moreover, as regards to the longevity-indexed life annuities with a profit share mechanism, the annuity prices represent between 92.51% and 102.24% of the price of an immediate fixed annuity. This means that for a reduced level of risk (λ=0.1) the financial returns compensate the losses generated by longevity changes', providing a better solution to the policyholder when compared with a traditional annuity. This percentage reduces with the increases noted in age and also in the risk parameter λ. For instance, for an individual with 50 years, and considering a λ of 0.2, the price of a longevity-indexed life annuity with profit share represents around 99.78% of the price of an immediate fixed annuity, whereas for a 75-year-old individual this percentage decreases to 96.25%. In addition, for the age of 50, if with a λ of 0.1, the price of a LILA is around 102.24% of the price of an immediate fixed annuity, when λ increases to 0.3, it is only around 97.71%.

	50	55	60	65	70	75
Immediate Fixed Annuity	35.76	30.72	25.84	21.12	16.66	12.55
LILA without profit share						
λ=0.1	34.77	29.78	24.95	20.32	15.94	11.93
λ=0.2	33.92	28.98	24.21	19.64	15.33	11.41
λ=0.3	33.22	28.31	23.59	19.07	14.82	10.96
LILA without profit share as a % of the Immediate Fixed Annuity						
λ=0.1	97.23%	96.94%	96.56%	96.21%	95.68%	95.06%
λ=0.2	94.85%	94.34%	93.69%	92.99%	92.02%	90.92%
λ=0.3	92.90%	92.15%	91.29%	90.29%	88.96%	87.33%
LILA with profit share						
λ=0.1	36.56	31.34	26.28	21.42	16.83	12.63
λ=0.2	35.68	30.50	25.50	20.71	16.20	12.08
λ=0.3	34.94	29.80	24.85	20.11	15.66	11.61
LILA with profit share as a % of the Immediate Fixed Annuity						
λ=0.1	102.24%	102.02%	101.70%	101.42%	101.02%	100.64%
λ=0.2	99.78%	99.28%	98.68%	98.06%	97.24%	96.25%
λ=0.3	97.71%	97.01%	96.17%	95.22%	94.00%	92.51%

Table 4 – Longevity-index life annuities premiums by age

Source: Author's elaboration.

Moving to Table 5, the results show that, as expected, the addition of a limit in the risk transferred to the policyholder causes an increase in the annuity value, meaning that the higher is the cap, the higher will be the annuity value and the protection of the policyholder. Overall, with a cap of 80%, the price of a capped longevity-indexed annuity represents between 94.50% and 102.24% of the price of an immediate fixed annuity. For a cap of 90%, this interval is between 98.01% and 102.40%. Analyzing the age of 50, it can be noted that, for a λ of 0.3, the annuity price increases from 34.94 to 35.21 with a cap of 80% and to 35.73 with a cap of 90%. Furthermore, for the same age and λ, the price of a Longevity-indexed annuity with a cap of 90% represents around 99.92% of the price of an immediate fixed annuity. In addition, when the limit imposed is 80%, this percentage decreases to 98.46%.

	50	55	60	65	70	75
Capped LILA with profit share - 90% Cap						
λ=0.1	36.62	31.40	26.34	21.48	16.89	12.68
λ=0.2	36.06	30.88	25.88	21.08	16.56	12.42
λ=0.3	35.73	30.59	25.63	20.88	16.40	12.30
Capped LILA as a % of the immediate annuity fixed annuity – 90% Cap						
λ=0.1	102.40%	102.21%	101.93%	101.70%	101.38%	101.04%
λ=0.2	100.84%	100.52%	100.15%	99.81%	99.40%	98.96%
λ=0.3	99.92%	99.58%	99.19%	98.86%	98.44%	98.01%
Capped LILA with profit share - 80% Cap						
λ=0.1	36.56	31.34	26.28	21.42	16.83	12.63
λ=0.2	35.76	30.58	25.58	20.79	16.27	12.15
λ=0.3	35.21	30.07	25.11	20.37	15.92	11.86
Capped LILA as a % of the immediate annuity fixed annuity – 80% Cap						
λ=0.1	102.24%	102.02%	101.70%	101.42%	101.02%	100.64%
λ=0.2	100.00%	99.54%	98.99%	98.44%	97.66%	96.81%
λ=0.3	98.46%	97.88%	97.17%	96.45%	95.56%	94.50%

Table 5 – Capped longevity-index life annuities premiums by age

Source: Author’s elaboration.

4.2.1.1. The utility-equivalent fixed life annuity

As referred in the methodology chapter, considering that the use of longevity-index annuities can harm the benefits’ amount that is provided to policyholders, it was also performed an analysis of the correspondent utility-equivalent fixed life annuity.

For this purpose, the utility was measured through a standard time addictive CCRA utility function defined over consumption. To obtain the utility levels, it was considered that the consumption for each individual equals the benefits provided by the correspondent LILA (Bravo, 2021).

Regarding the coefficient of relative risk aversion, three levels were considered, namely $\gamma = 2$ for low risk, $\gamma = 5$ for medium risk and $\gamma = 10$ for high risk. For the subjective discount factor, it was considered that the policyholder can be patient (β of 0.98), normal (β of 0.96) or impatient (β of 0.94). It was also assumed a λ of 0.3 and an interest rate of 0%. The results are presented in Table 6.

		50	55	60	65	70	75
β = 0.98	γ = 2	38.09	32.67	27.42	22.38	17.61	13.24
	γ = 5	37.21	31.82	26.62	21.63	16.95	12.68
	γ = 10	33.45	28.45	23.67	19.15	14.94	11.14
β = 0.96	γ = 2	38.88	33.35	27.99	22.83	17.95	13.47
	γ = 5	38.29	32.74	27.38	22.23	17.38	12.98
	γ = 10	35.41	30.02	24.88	20.03	15.56	11.55
β = 0.94	γ = 2	39.49	33.90	28.46	23.22	18.25	13.69
	γ = 5	39.11	33.48	28.01	22.74	17.78	13.26
	γ = 10	37.18	31.48	26.04	20.90	16.17	11.95

Table 6 – EFLA for the longevity-index life annuities with profit share
Source: Author's elaboration.

Overall, the results show that the EFLA value decreases as the level of risk aversion increases. For example, a 50 years patient annuitant with a low level of risk aversion (β=0.98, γ=2), the EFLA equals 38.09, whereas for an individual with the same age but with a high level of risk aversion (β=0.98, γ=10), the value of the EFLA drops to 33.45.

Furthermore, for individuals of all ages and with low or medium risk aversion levels, the EFLA is always higher than the value of the correspondent immediate fixed annuity. For instance, an impatient individual with 55 years and a medium risk aversion (β=0.94, γ=5), has an EFLA of 33.48, which compares with a price of 30.72 for the correspondent immediate fixed annuity (see Table 4). This means that even with a possible reduction of the benefits caused by changes in longevity, low and medium risk-averse individuals would be willing to pay more for a longevity-index life annuity that would allow them to take advantage of a profit share mechanism.

However, for the majority of the high risk-averse individuals, it is observed the opposite result. In fact, for a 60-year-old policyholder with a β of 0.96 and a γ of 10, the EFLA is around 24.88, while the value of the correspondent immediate fixed life annuity is around 25.84 (see Table 4). This effect is observed for patient and normal individuals of all ages, and for impatient individuals with ages between 65 and 75 years old.

4.2.2. Longevity-contingent deferred life annuities

This section presents the annuity premiums obtained as regards to the longevity-contingent deferred life annuities, including the impacts of changes in the risk parameter λ. Similar to the previous case, it was considered an interest rate of 0%.

As explained above, in this type of annuity, an initial period of deferment (d) is defined in the beginning of the contract. Once, the annuitant reaches the age $x_0 + d$, the life expectancy is recalculated with the new data available and, if, above a certain threshold, the payment is deferred for an additional period (k). The additional period of deferment will end whenever the life expectancy reaches the threshold.

To apply this framework, it was assumed that, for each age x, d is the number of years until the retirement age (as performed by Denuit et al., 2015). Considering that, as of today, the retirement age in Portugal is around 66.6 years, it was assumed a retirement age of 67 years. This means that, for example, an individual with 50 years at the beginning of the contract would have an initial deferment period of 17 years. Taking into account this assumption, the study of this contract structure was focused on ages between 50 and 65.

In terms of the threshold that determines the extent of the deferment period, it was considered that for each age, the payments start when the life expectancy of an individual aged $x_0 + d + k$, calculated with the application of a Wang Transform, is smaller or equal to the best estimate of the life expectancy at the age of 67.

The calculations performed, tested values of k between 1 and 10, for the different ages and values of λ. As expected, the results presented in Table 7, show that as λ increases, the additional period of deferment needed to reach the threshold defined in the contract also increases, reflecting the higher levels of life expectancy. Overall, the results show that for the different ages and risk levels, the additional period of deferment ranges from 1 to 3 years. For example, for the age of 50, after an initial deferment of 17 years, with the lower level of risk, the annuitant would have to wait for an additional period of 2 years, until the beginning of the payments, whereas, if the level of risk is 0.3, the same individual would have to wait for 3 years.

However, it should be noted that an individual may not be willing to wait for a long period until the payments start. Therefore, imposing an additional period deferment that is too large threatens the potential demand for this type of annuity. As a result, the insurer may consider a cap, limiting k to a maximum value (Denuit et al., 2015).

To consider this feature, caps of 2 and 1 years were tested. The consequence of these tests is of course an increase of the annuity value for the different ages. This increase in the annuity premiums ranges from 5.46% to 5.91% for a 2-years cap and from 5.75% to 11.94% if a 1-year cap is considered. For instance, for an individual with 50 years and a λ of 0.2, the annuity value would increase from 17.07 (20 years of total deferment) to 18.05 if k limited to 2 years (19 years of total deferement) or to 19.05 if k limited to 1 year (corresponding to 18 years of total deferment). Therefore, for the age of 50 and a λ of 0.2, the application of these caps would result in an increase of 5.73% and 11.64%, respectively.

	Additional Deferment period (k)	Total Deferment period ($d+k$)	Annuity value (1)	Annuity value 2 years cap (2)	% of variation between (1) and (2)	Annuity value 1 year cap (3)	% of variation between (1) and (3)
Age 50, initial deferment of 17							
λ=0.1	2	19	16.93	16.93	0.00%	17.92	5.90%
λ=0.2	3	20	17.07	18.05	5.73%	19.05	11.64%
λ=0.3	3	20	18.15	19.14	5.46%	20.15	11.06%
Age 55, initial deferment of 12							
λ=0.1	2	14	16.80	16.80	0.00%	17.82	6.08%
λ=0.2	2	14	17.89	17.89	0.00%	18.92	5.75%
λ=0.3	3	15	17.95	18.95	5.61%	19.99	11.37%
Age 60, initial deferment of 7							
λ=0.1	1	8	17.95	17.95	0.00%	17.95	0.00%
λ=0.2	2	9	17.94	17.94	0.00%	19.00	5.91%
λ=0.3	3	10	17.93	18.96	5.77%	20.02	11.68%
Age 65, initial deferment of 2							
λ=0.1	1	3	18.47	18.47	0.00%	18.47	0.00%
λ=0.2	2	4	18.31	18.31	0.00%	19.42	6.06%
λ=0.3	3	5	18.18	19.25	5.91%	20.35	11.94%

Table 7 – Longevity-contingent deferred life annuities: premiums and deferment periods. Source: Author’s elaboration.

5. CONCLUSIONS

Over the last decades, the advances in medical care, the increasing concerns with the adoption of a healthy lifestyle, among other factors, have been leading to an increase of the life expectancy of the populations, exposing individuals and insurers to systematic longevity risk. In fact, for the Portuguese population, an increase of the life expectancy at birth around 2 years has been verified in the last decade. Furthermore, the life expectancy at 65 years as also shown a positive evolution for both men and women (INE, 2020).

For individuals, this means that there is a possibility of outliving their assets which can be subject of concerns due to the low level of benefits provided by the Portuguese Social Security System that is currently facing the risk of becoming unsustainable (Olivieri & Pitacco,2019; Bravo et al., 2013, 2014; Moreira et al.,2019). On the insurer's side, the difficulty in predicting life expectancy leads to issues in terms of the guarantees provided to policyholders, increasing the capital requirements.

To create a solution for this issue in the insurance market, several alternatives have been studied in terms of product design that would allow a higher control of the longevity risk. At this point, the insurers can take two different approaches: a complete transfer of the longevity risk to the policyholder; or the creation of risk-sharing mechanisms between the company and the individuals.

To consider both the point of view of the insurer and the point of view of the individual, this investigation was focused on products with risk-sharing mechanisms. Therefore, two types of contract structures/products were selected to test their impact on the Portuguese context in terms of pricing. The products selected include: Longevity-index life annuities (Denuit et al., 2011; Bravo & El Mekkaoui, 2018); and Longevity-contingent deferred life annuities (Denuit et al., 2015).

The methodology followed was based in three steps, namely: i) the projection of age-specific mortality rates through the Poisson Lee-Carter Model and the application of a Wang Transform to consider the market price of the longevity risk; ii) the projection of the financial results, through the application of a Geometric Brownian Motion for stocks and of a 2-factor Vasicek model for bonds; iii) and finally the application of a standard time addictive constant relative risk aversion model to measure the EFLA for the Longevity-index life annuities.

Concerning the Longevity-index life annuities, the results obtained show that, overall, the higher the market price of longevity risk, the lower will be the annuity price. Furthermore, when no limit is imposed on the risk transferred to the individuals, the price of a LILA with profit share represents between 92.51% and 102.24% of the price of an immediate life annuity, meaning that, in a low-risk scenario, for certain ages, the losses created by longevity changes are compensated by the financial returns included in the contract. In addition, when a cap on the risk transferred to individuals is imposed, the results obtained show an increase of the annuity value thus enhancing the protection of the policyholder. Overall, with a cap of 80%, the price of a capped LILA represents between 94.50% and 102.24% of the price of an immediate fixed annuity. For a cap of 90%, this interval is between 98.01% and 102.40%.

In terms of the utility to the policyholder, it was verified that for all of the low and medium risk-averse individuals', the EFLA values are always higher than the correspondent immediate fixed annuities. Still, for the high risk-averse individuals it was obtained the opposite result for most of the ages under analysis.

As regards to the Longevity-contingent deferred life annuities, where the period of deferment varies following changes noted in life expectancy, considering a threshold defined at the contract inception, the results obtained show that, as expected, the higher is the market price of longevity, the higher will be the deferment period needed to manage the risk. Overall, considering that there is an initial deferment that ends at the retirement age, the results show that for the different ages and risk levels, the additional period of deferment needed to reach the life expectancy threshold defined ranges from 1 to 3 years. In addition, caps on the maximum deferment period were tested, resulting in an increase of the annuity value for the different ages. This increase of the annuity premiums ranges from 5.46% to 5.91% for a 2 years cap and from 5.75% to 11.94% if a 1-year cap is considered.

Finally, it should be highlighted that the use of longevity-linked annuities provides a relevant solution to manage the insurers' concerns with unexpected changes in life expectancy, while also gives the individuals a chance of acquiring an annuity product at a reduced price. Nevertheless, considering the risks taken by the individuals with a possible reduction of the benefits level or the extension of the deferment period, the implementation of caps should always be considered by the insurance companies to deal with eventual issues in terms of demand of this type of products.

Considering the extension of the literature and the wide range of contract structures being suggested, future investigations could be focused on the implementation of different alternatives in terms of product design into the Portuguese reality, such as the use of mechanisms where the adjustment method is based on the reserves (Maurer et al., 2013, among others). Furthermore, it would also be interesting to perform a study on the demand for longevity-linked annuity products in the Portuguese market.

To conclude, it should be noted that the impacts on the COVID-19 pandemic in the Portuguese mortality rates were not considered in the estimations made in this investigation, since they are still not completely known.

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